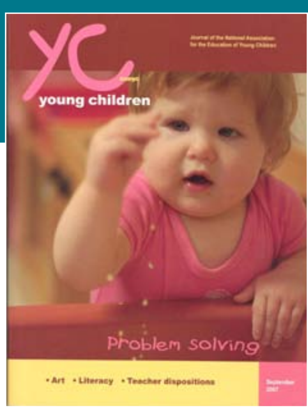
Developmentally Appropriate Practice

A reading from the CD accompanying *Developmentally Appropriate Practice in Early Childhood Programs Serving Children from Birth through Age 8, Third Edition.*

READING #11

The Classroom That Math Built: Encouraging Young Mathematicians to Pose Problems



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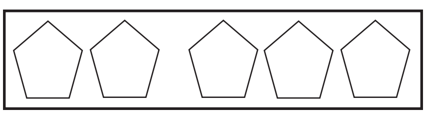
The Classroom That Math Built

Encouraging Young Mathematicians to Pose Problems

Ann H. Wallace, Deborah Abbott, and Reneř McAlhaney Blary

The idea of problem posing is not new, but it has received increased attention in light of new approaches to mathematics education. The National Council of Teachers of Mathematics (NCTM 1991, 2000) has called on teachers to set up problem-solving experiences that encourage children to devise and solve their own problems.

Mathematical problem posing builds on young children's natural curiosity. It promotes children's "engagement in authentic mathematical activity"; enables children to "encounter many problems, methods, and solutions rather than only one of each"; and fosters creativity (Silver & Cai 2005, 129). For example, children may see an illustration of pentagons in a book or worksheet. The implied question is, "What is 2 + 3?" However, the picture can be used as a vehicle for additional problem posing. Various children may ask the following questions:



- If these are all tables, how many guests could sit at each table? How many guests could you seat at all the tables?
- If there are two adults at each table, how many places are left for children?
- If 16 seats are filled, how many seats are empty?
- Why is there a gap? How many shapes would fill the gap?

Some teachers find it difficult to let children pose and explore their own problems. They may assign textbook problems that often are of little interest to the class and that generally have one solution reached only after following a particular procedure (Baxter 2005). Traditionally it was the teacher's role to lead students down that specific path to the solution.

But real things are not so neatly packaged in the natural world, where there are no set procedures or pat answers. Early educators must move away from posing math problems with one way to reach a solution to offering math experiences with things and situations that are meaningful to children. As children explore their environment, they ask and pose questions. Effective teachers recognize and provide opportunities and experiences to build math skills and concepts within that environment. One way to do this is by guiding classroom discussions about children's problem-solving experiences.

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Photos courtesy of the authors.

Teacher input into the problem-posing process is critical. Their questions frame the concept or idea available for exploration, analysis, and consideration (Whitin 2006). In the previous example, a child referred to the pentagons as tables. The teacher then pursued this idea and encouraged children to pose problems as if that were the case. This gave a context to children who may have been struggling with their own problems to pose. In classroom environments that encourage and support exploration, problem solving becomes important to children. They can think about, analyze, and solve problems that matter to them.

This article describes how one teacher encouraged children to pose their own math problems during a five-day investigation in their classroom. Included are examples of teacher questions and children's posed problems and solutions.

Open-ended class discussions and encouragement are integral parts of the learning process.

Deborah's class

Second-grade teacher Deborah Abbott, one of the authors of this article, consistently strives to create a classroom environment in which she and the children share and support the construction of mathematical thinking. Her goal is to promote critical thinking across all disciplines. To enhance hands-on, authentic learning activities, she stocked the classroom with multiple and varied math resources, including manipulatives such as two- and three-dimensional geometric shapes, linking cubes, counters, number tiles, and pattern blocks; tools such as measuring tapes, rulers, calculators, graph paper, and writing supplies; and books focusing on authentic use and real-world math experiences specific to a topic or project—for example, for a study of the community, *Numbers on the Street* (Math All Around series), by Jennifer Rozines Roy and Gregory Roy, and *Shapes around Town*, by Nathan Olson.

Open-ended class discussions and encouragement are integral parts of the learning process. Children have opportunities to recognize, verbalize, explore, and evaluate in an environment that fosters the problem-solving process and where teacher and students alike ask and answer questions.

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Deborah wanted to incorporate problem posing into her teaching. Its open-ended process fit her philosophy of providing opportunities for children to investigate multiple approaches to solving problems. She had never thought about setting up an initial problem to spawn new questions and was curious as to whether her second-graders could create their own extensions from a problem situation. She planned a five-day investigation based on inquiry and process skills.

Deborah knew she could not just begin with, "Here is a problem. What question do you want to pose?" and expect the children to respond. Any initial problem situation had to interest and make sense to the children or they would not become engaged. Deborah speculated that "if my students had to determine how many hamburgers were eaten in one day at school, they would love it!"

After deciding to focus and build on something the children saw every day—their classroom—she asked herself, What do the students notice about this classroom? If they had to create their own, could they? What would their classroom look like? She wondered if the children could

- realize or recognize the size of classroom items (tables, bookcases, rug, and so on),
- identify the arrangement of objects,
- comprehend amounts (like the number of books or blocks), and
- understand the mathematics involved in finding out these things.

To get students thinking about everyday numbers—where we see them and how we use them—Deborah asked the children questions like the following:

- Our school is filled with math. Where do you see numbers here?
- At home, in the car or store, where do you see numbers?
- How do we use these numbers every day?
- How do you use the numbers you see in our school?

A lively discussion began. The children mentioned the cost of lunch, cooking cups, calendar days, and telling time. This was just a brainstorming and preliminary session, so the class did not keep a record of the everyday numbers.

Day 1: Building interest

To help students make connections between mathematics and their world, Deborah introduced *The House That Math Built,* from Time-Life Books, as a context for the problem-posing investigation. The book highlights the importance of early math skills through original stories, poems, riddles, games, and hands-on activities. It encourages children to see math as a natural part of their everyday environment. The book takes readers on a tour of a wacky mansion filled with math challenges. Every room has a theme, like the Shape Room and the Measuring Room. Using the book's suggestions for extending the focus of each page, Deborah prepared additional questions to ask as she read. For example, in the Pattern Bathroom, she asked the children:

- What color is the missing wallpaper flower?
- What is the pattern on the shower curtain?
- Can you extend the pattern?

Then Deborah posed the question, "What if the next room we walked into was a classroom? I wonder what it would look like." She asked the children to close their eyes and use their mind's eye to see what the classroom would look like. She asked questions that helped create visual images:

- What would you put in your classroom?
- How many would you need?
- Where would things go?
- Do you need exact numbers of things?
- Does it matter?

"What if the next room we walked into was a classroom? I wonder what it would look like."

Children shared their ideas. "I see a lot of desks," one said. "Kids!" said another. "Books!" "Book bags," and more. Deborah then wondered about drawing the imaginary classroom. "Hmm," she said, "How would we begin such a drawing? Would we have to plan first? What things do you think we should include?"

The thorough list included tables, chairs, calendar, computers, bulletin boards, books, posters, tissue boxes, and many more items.

Deborah asked the children to draw diagrams of the classrooms they visualized. She walked around, observing and commenting on the children's work. She reminded everyone to think about what items would be needed in the room. She often asked probing questions:

- Do you think that's enough?
- Will the teacher need different materials than the students?
- Does the teacher need more or fewer of certain items?
- Do you think it will all fit?
- How can you find out?
- What things does your classroom have to have?
- Do you think you're finished?
- Are you satisfied?

Her many questions were designed to assess whether every child could understand and interpret the problem.

Day 2: Introducing a problem

The children gathered to share and discuss the classroom diagrams they had drawn the previous day. Together they generated a master list of items they decided were needed for a classroom, noting things they had not included and considering other suggestions. The thorough list included tables, chairs, calendar, computers, bulletin boards, books, posters, tissue boxes, and many more items.

"Now," said Deborah, "if you were going to design your own classroom—let's call it 'The Classroom That Math Built'—are these the items you would need?" She had to figure out how to transition from the generated list to further exploration, encouraging the children to ask questions they could answer. Knowing that this would be a challenge, she had planned in advance how to get the children thinking about quantity. She prodded,

- Is the size or the amount of the items important?
- Does length matter?
- Would it matter how many desks we had? Could they be too big or too small?
- How big is the carpet? Why do we care how big it is?
- How can we answer these questions?

Referring back to the list of items, she asked, "Can we turn some of the things on our list into problems or questions?" One child responded, "I'd like to know how long the shelves are." "How do we figure that out?" Deborah asked. Several children replied in unison, "Measure!"

Deborah followed up: "What tool would we use to measure the shelf?" She asked one child to choose the best measuring tool for determining the length of the shelf. The boy picked up both the yardstick and the measuring tape, pondered a moment, and then decided on the tape. Deborah inquired, "Why did you choose the measuring tape over the yardstick?" He responded, "There's more and it's easier. I just have to roll out the tape one time, and I don't have to start and stop, like with the yardstick."

The class created a list of available measurement tools—rulers, yardsticks and meter sticks, measuring tape, calculators. They also mentioned scales but decided it was a measuring tool they probably would not need.

The children then selected at least one item from the list and worked with partners to generate a question to ask about the item(s). Deborah provided a few constraints:

- The room must be the same size as their real classroom.
- There would be no extra windows or doors.
- The classroom would have one teacher and 20 students.

By the end of math class, the children had discussed a variety of items and the tools with which to measure them. They were ready to begin solving their problems.

A Five-step Problem-solving Process

1. **Understand the problem.**
   What is your question? Do you know what you need to do? Do you know what you're looking for? Do you count or measure? Do you need to record things? Do you need to make notes? Can you explain this to your partner?
2. **Devise a plan.**
   Do you need to draw a picture? What tools do you need? Share your thinking with your partner. Do you agree?
4. **Carry out the plan.**
   Do you need a record of your investigation? How will you keep a record?
4. **Answer the question.**
   Do you have an answer to the question you posed? Is there anything else you would like to know?
5. **Does your answer make sense?**
   Check your work. Are 200 desks correct? Is the carpet way too big? Why is it important to check our work?

Adapted from George Polyá, *How to Solve It: A New Aspect of Mathematical Method*, 2nd ed. (Princeton, NJ: Princeton University Press, [1957] 2004).

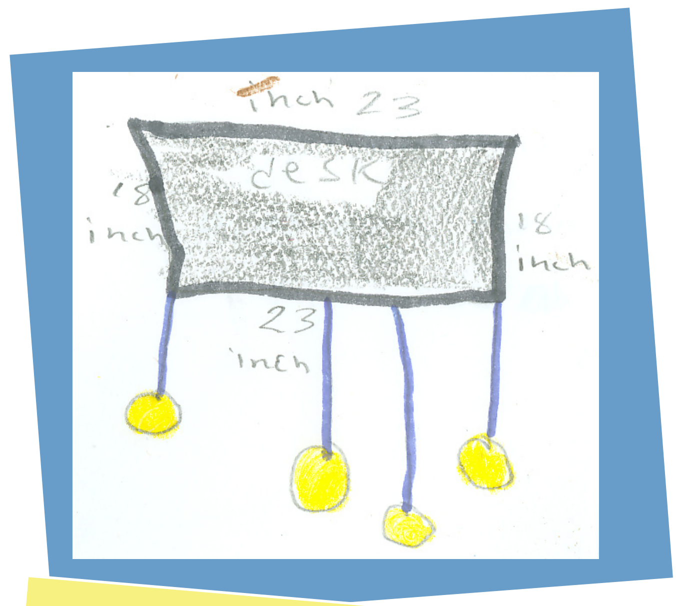
Day 3: Let the problem solving begin

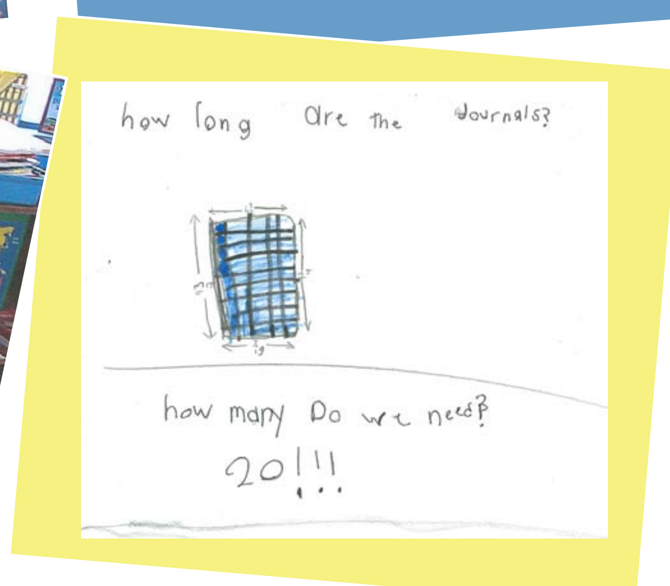
Deborah reviewed an adaptation of a classic problem-solving process (Polyá [1957] 2004) that the children had previously used (see “A Five-step Problem-solving Process,” p. 45). While going over each step, she asked questions to help the children focus on the questions they had posed.

With their chosen tools, the children measured and talked about solving their problems for the remainder of the math period. They tallied, counted, and calculated numbers all over the classroom, measuring the perimeter of rugs and the length, width, and height of the bulletin board, bookcases, tables, and other things.

Deborah was pleased with their serious approach to problem solving. She saw that the problems were important to the children and that they were taking an active approach to solving them. As she observed the work, she asked questions to examine and extend the children’s thinking and to help them connect mathematical ideas. To show respect for their ideas, however conventional or unconventional, she listened to various approaches and possible solutions. Children worked with their partners and only turned to Deborah for help or direction when necessary.

For the question, “How many desks and chairs do we need for the 20 students in the class?” Deborah guided problem-solving partners by asking additional questions, such as “Do you need exactly 20? What would happen if we got a new student? Would the student have to sit on the floor? What size desks do you think we need?”





With another team investigating how many bookcases the classroom would need, Deborah facilitated their thinking by asking, "What size should they be? How many shelves are needed? Do the shelves have to be the same size? Will your journals fit on one of the shelves?"

The children became so involved that they wanted to continue beyond math time.

Day 4: Sharing

Each pair or group of students shared with the class the problem they had solved, their method(s) for solving it, and the answer(s) they had found. Deborah occasionally asked questions to foster sense making and promote reasoning. She structured the flow of discussion so that everyone in the class could understand the explanations. For example, she asked,

- Would anyone have answered that question in a different way?
- How could you improve your methods?
- How could you check your results?
- Would checking be important?
- If you didn't have the tools you had, what would have happened?
- What other tools would have been helpful?

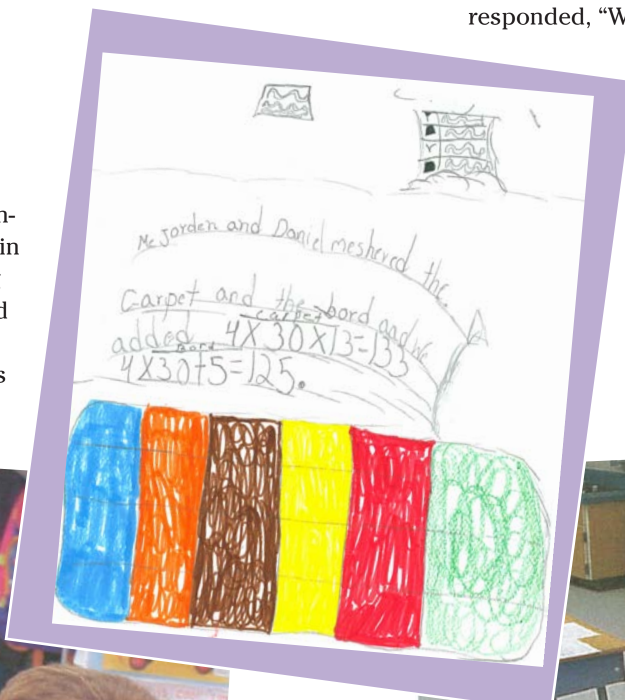
The class concluded that recording their work to communicate their findings was very important. Deborah wondered, "Could you measure things again if you were asked? Why was recording important?" Although the children had used pencils, paper, and calculators, they said they thought that computers could be used for bigger projects.

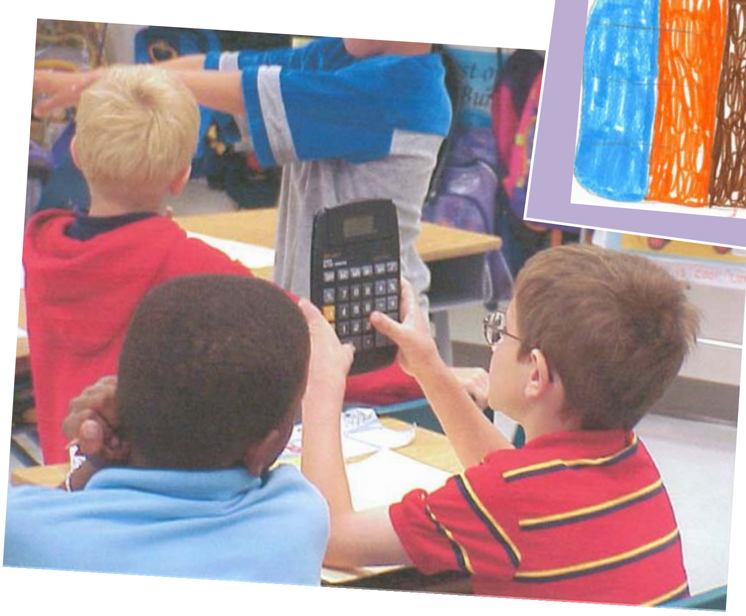
Day 5: Reflecting

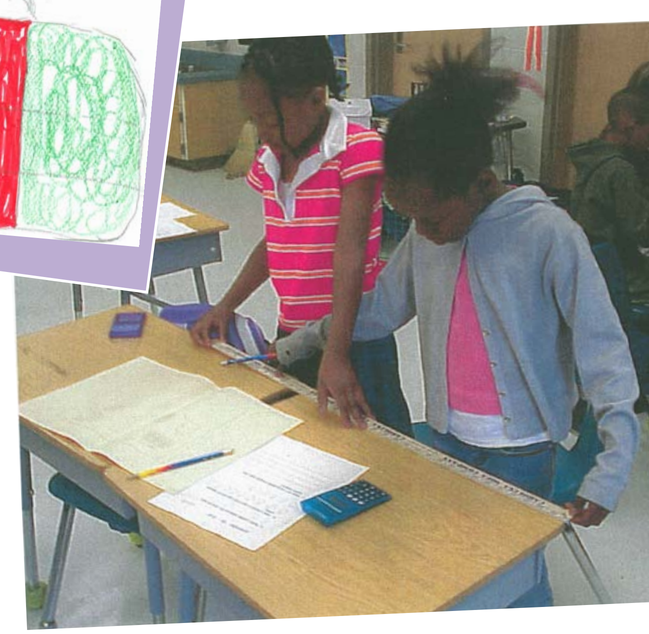
On the final day of problem posing, Deborah and her clinical intern interviewed each pair of children. She asked them to describe their thinking process for the question(s) they had answered. When she asked David and Cody what materials they had used to determine the perimeter of the rug, David replied, "Well, we used four yardsticks and we used a 12-inch ruler. We had to put the rulers together to get the exact length." Cody added, "First it was 120—it said so on the calculator—and then David said that's the wrong answer, and then I said it's 133." When asked how they had reached their answer, David explained, "We had to use the rulers plus the markers to mark our areas. I counted how many marks we made. I came to the conclusion by adding up this number sentence: 4 x 30 + 13 = 133."

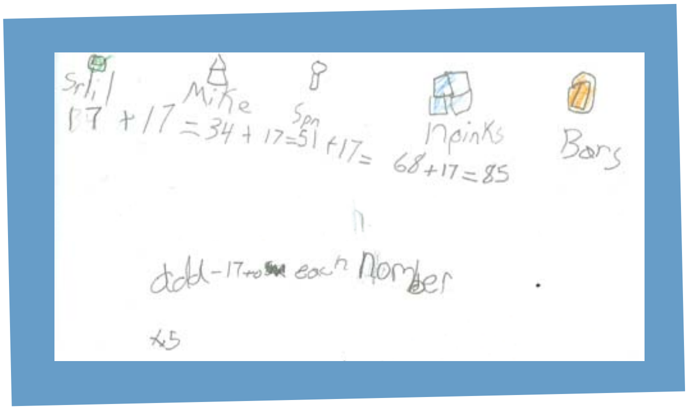
Alexis and Diamond had measured the perimeter of their desks put together. Alexis reported, "We took the ruler and then we put it on the desk and we measured how much around." Diamond clarified, "We wrote on a piece of paper so that we can remember what we got. We counted a little bit. Like we used the pencils, so that it could help us, as we get to a point we can put a mark down." When asked, "Were there things you didn't find out that you wanted to?" Alexis responded, "We can measure other things that I want to know, like the whole classroom, like big stuff—bigger than this desk or bigger than the board."

Bethany, working by herself, had asked the unconventional and ambitious question, "What do we need for everyone to eat breakfast?" For a class of 17, she had determined









that 17 cereal bowls, 17 cartons of milk, 17 spoons, 17 napkins, and 17 cereal bars would be needed. She added 17 + 17 = 34, 34 + 17 = 51, 51 + 17 = 68, 68 + 17 = 85. She had concluded that the class would need 85 items for breakfast. When Deborah asked Bethany why she had used 17 when there were 20 students in the class, Bethany responded that three children did not eat breakfast. When asked if she could use the same method to solve a different problem, she said, "I guess—pencils or batteries or clipboards."

Bethany wasn't done. She said she wanted to figure out how many school supplies would be needed for 85 people. (She chose the number *85* from the number of breakfast items she had previously found.)

Lessons learned

This five-day experiment reinforced the importance of a classroom environment that promotes mathematical thinking. Such a classroom is led by a teacher who is flexible and eager to listen to what children have to say, and who accepts their ideas.

Deborah says the experience served the needs of a wide variety of mathematical learners. It opened math gateways for children with differing intellectual needs and desires. In expressing their math ideas, some children drew, some wrote, some tallied, some recorded, some created graphs, and so on.

There was David, the self-proclaimed class mathematician, and his partner Cory, who was still struggling with one-to-one correspondence. The vast difference in their mathematical skills did not deter them from exploring and solving their problem.

Deborah understood the importance of capturing the children's interest before posing the problem she wanted them to explore. She began with the engaging story of the wacky mansion and followed with a series of questions. Only after she had piqued the children's interest did she introduce the focus for the lesson, the classroom. As a result, the children relied less on her direction and more on themselves to come up with problems to be solved. She further found Polyá's ([1977] 2004) problem-solving process helpful in guiding students in what to do with the questions they posed.

Although the children did not extend their problems by posing additional questions, Deborah believes her classroom culture invites this kind of expansion. She suspects the children had little or no prior experience in posing problems, and she thinks the initial challenge, imaginary classrooms, may have been too narrow. There might have been further explorations if she had transitioned from focusing on the classroom to exploring other places of interest to the children (like their bedrooms or the zoo or mapping the neighborhood).

In the future, Deborah plans to incorporate problem posing throughout the year, perhaps by enlisting older children to help the second-graders with math exploration. The older children may bring different ideas about what problems to pose or questions to ask. She will continue to cultivate the classroom environment by creating positive mathematical experiences for her students.

Conclusion

This experience demonstrates the value of allowing children to take responsibility for their own learning and emphasizes the crucial role the teacher plays in this process. Effective early childhood educators teach children how to think. The goal of problem posing is for children to figure out how to ask a question from or about a given situation.

The children in Deborah's class had to figure out what to do when they were not being told what to do. They relied less on the teacher's questions and answers and become comfortable not following a predetermined process. They talked through problems to figure out other problems. They speculated, pursued alternatives, and determined whether their approaches were valid. They discarded ideas that did not work and focused on those that did. As Deborah says, "These are the essential skills they will need for their future success in the world."

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